1 Example 2.14(b)

The problem was as follows. There are offices in Atlanta, Baltimore, Chicago, Denver, Rochester, Seattle and Tampa, and we need to visit 4 different offices. A schedule is a set of 4 different offices to visit and an order in which to visit them. How many schedules include visits to Chicago or Denver or both?

In class, we considered the complement of "include Chicago or Denver or both". That is, we considered schedules that do not include Chicago and Denver and found the answer.

Can we solve the problem without take complement? The answer is yes.

How? There are two cases need to be considered: (a) Chicago is included; (b) Denver is included but Chicago is not. (In class, I said that there are three cases, (i) Chicago is included but Denver is not; (ii) Denver is included but Chicago is not; (iii) both Chicago and Denver are included. This is more complicated, so I decided to divide it into only two cases (a) and (b) instead of (i), (ii) and (iii).)

First consider case (a), Chicago is included. Chicago can be the first, second, third or fourth office we are going to visit. If it is the first office, then we will see something like

$$\underline{\text{C}} \quad \_ \quad \_ \quad \_$$

The second slot can be any city other than Chicago, so there are 6 possible choices. Similarly, the third slot has 5 choices and the last slot has 4 choices. In this case we will have $6 \times 5 \times 4 = 120$ many schedules.

But Chicago can also be in the second slot, the third slot or the fourth slot:

$$\_ \quad \underline{\text{C}} \quad \_ \quad \_$$

$$\_ \quad \_ \quad \underline{\text{C}} \quad \_$$

or

$$\_ \quad \_ \quad \_ \quad \underline{\text{C}}$$

These can be done similarly, and we will see that there are altogether $120+120+120+120 = 480$ many schedules for case (a).

How about case (b), Denver is included but Chicago is not? It is similar but slightly more complicated. Suppose that Denver is in the first slot. So we will have

$$\underline{\text{D}} \quad \_ \quad \_ \quad \_$$

Now, the second slot can be any city other than Denver and Chicago. Therefore, there are only 5 cities for us to choose in the second slot, and hence 4 cities for the third and 3 cities for the last. Altogether there are $5 \times 4 \times 3 = 60$ possible schedules.

Again, Denver can be in the other slots, and so there will be $60 + 60 + 60 + 60 = 240$ many schedules for case (b).

Adding the number of schedules for both cases (a) and (b), we see that there are $480 + 240 = 720$ many schedules including Chicago and Denver.

2 Remarks

In this case, we see that it is still possible to solve this problem without taking complement. But what will happen if you are asked to find the number of schedules that include Baltimore, Chicago or Denver (or any two of them or all of them)?

Notice that there are many more cases we need to consider, which is time consuming and it will be easier for you to make mistakes. In contrast, if we consider its complement, we just need to consider the schedules that do not include all those three cities, so what we need to do is just arranging 4 cities from the 4 remaining cities, and there are $P(4,4) = 4\times3\times2\times1 = 24$ schedules, and so there are $840 - 24 = 816$ many schedules that include Baltimore, Chicago or Denver (or any two of them or all of them).